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A nonstandard boundary layer in time in the study of correctors for a parabolic equation in a thin heterogeneous structure

We study the correctors problem for a parabolic equation posed in a thin heterogeneous structure which is a parallelepiped $\Omega_\varepsilon = \big(-\dfrac{\varepsilon}{2}, \dfrac{\varepsilon}{2}\big)^2 \times (-\dfrac{1}{2}, \dfrac{1}{2})$, where ε is a small parameter. The structure is heterogeneous in the sense that it contains a fiber $F_\varepsilon = \varepsilon D \times (-\frac{1}{2}, \frac{1}{2})$ (D is the disk $D := D(0, r), 0 < r < \frac{1}{2}$) whose conductivity coefficient is equal to 1 while the conductivity coefficient of the material surrounding the fiber, which occupies the region $M_\varepsilon = \Omega_\varepsilon - F_\varepsilon$, is equal to ε^2.

Using a classical scaling, we first investigate the limit problem (as ε tends to zero) following the method developed in [2] and then, we show that the solution u_ε of the corresponding singular perturbations problem may be written as $u_\varepsilon = \tilde{u}_\varepsilon + \hat{u}_\varepsilon$, where $\tilde{u}_\varepsilon$ is the solution of a problem with special data (the corrector for u_ε) and where $\hat{u}_\varepsilon$ appears as a time boundary layer concentrated at the time origin in the case of a parabolic term of the form $\big(\chi_{F_\varepsilon} + \varepsilon^2 \chi_{M_\varepsilon}\big)\frac{\partial}{\partial t}$ and as a perturbation which remains beyond the time $t = 0$, in the case of a parabolic term of the form $\dfrac{\partial}{\partial t}$.

Moreover, in the framework of periodic homogenization, i.e., in the case of several such structures Ω_ε^i, $i \in I_\varepsilon$, periodically distributed in a cube Ω with a period of size ε, such a phenomenon remains even if one deals with a parabolic term of the form $\big(\chi_{F_\varepsilon} + \varepsilon^2 \chi_{M_\varepsilon}\big)\frac{\partial}{\partial t}$. This phenomenon does not arise in classical homogenization of parabolic equations since in that case, the time boundary layer is always concentrated at the time origin $t = 0$, (see [1]).

[1] S. Brahim-Otsmane, G. A. Francfort, F. Murat, Correctors for the homogenization of the wave and heat equations, *J. Math. Pures Appl.* **71** (1992), 3, 197-231.

[2] A. Sili, Homogenization of a nonlinear monotone problem in an anisotropic medium, *Math. Models Methods. Appl. Sci.* **14** (2004), 3, 329-353.