Björn Gustafsson

Homogenization and reduction of dimension in linear elasticity

I will report on some recent joint work with Jacqueline Mossino. It concerns the adaptation of a decomposition technique, developed by Courilleau, Dufour, Fabre and Mossino for conductivity matrices, to the case of elasticity tensors.

For a linear elastic material the relationship between the stress tensor $\sigma = (\sigma_{ij})$ and the strain tensor $e = (e_{kl})$ is of the form

$$\sigma_{ij} = A_{ijkl} e_{kl},$$

or briefly $\sigma = Ae$, where $A = (A_{ijkl}(x))$ is the elasticity tensor. We consider the case of a stratified material and show how one can treat simultaneous homogenization and reduction of dimension by writing the above relation as

$$M\sigma = Pe$$

for appropriately chosen tensors $M = (M_{ijkl})$ and $P = (P_{ijkl})$ and by passing to the limit (all tensors above depend on some fine structure parameter ε) using compensated compactness applied to each member of the equation.

A preprint version of the corresponding detailed paper, "B. Gustafsson, J. Mossino, Compensated compactness for homogenization and reduction of dimension: the case of elastic laminates", is available at

http://www.math.kth.se/ gbjorn/