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Composite laminates: modeling and computational aspects

Composite materials find great interest in a variety of complex structures, such as those typically adopted in space, automobile and civil applications. The design of these structures clearly requires numerical tools able to perform accurate stress analysis. As a matter of fact, nowadays many commercial finite-element codes contain laminated composite plate and shell elements. However, the proper modelling of laminated plates is a non-trivial task and it can be still considered as an open research problem mainly as a consequence of their complex behavior.

Composite laminates have an anisotropic response, present significant shear deformation in the thickness and extension-bending coupling. Furthermore, the determination of accurate values for the interlaminar normal and shear stresses is of crucial importance, since they are responsible for the activation and the development of delamination mechanisms. A satisfactory laminate theory and a corresponding reliable finite-element have to capture all these effects.

Several laminate theories have been proposed in the literature. A review of laminated composite plates is proposed emphasizing the main aspect of the different theories. The first and simplest one is the classical laminate theory (CLT), based on the Kirchhoff-Love assumptions; the CLT neglects the shear deformation and, as a consequence, it appears inadequate to reproduce the behavior of composite laminates.

The extension of the Reissner and Mindlin models to the case of laminated anisotropic plates is of great interest for the possibility of taking into account shear deformation effects in a simple way.

The First-order Shear-Deformation Theory (FSDT), extension of the Reissner and Mindlin models to the case of laminated anisotropic plates, accounts for shear deformation effects in a simple way. It is able to lead to satisfactory results for a wide class of structural problems, even for moderately thick laminates.

Higher-order laminate theories have also been developed to overcome the limitations of the FSDT. Two different approaches can be distinguished: single-layer and multi-layer formulations.

The first class of theories increases the order of the terms considered for the displacement representation in the thickness coordinates. The second class of theories assumes a representation formula for the displacement field in each layer. From a computational point of view, several laminated plate finite elements are proposed in the literature. Two finite elements are briefly illustrated. The first finite element EML4 is based on a mixed-enhanced formulation and uses enhanced incompatible modes to improve the in-plane deformation, bubble functions for the rotational degrees of freedom and functions linking the transverse displacement to the rotations. The element is locking free, it does not have zero energy modes and is able to provide accurate in-plane and out-of-plane deformations. Then, an extension of the MITC (mixed interpolated tensorial components) plate elements to the case of composite laminates is proposed, presenting 4- and 9-node composite laminated MITC plate elements.

Finally, a very special problem is illustrated. It consists in the case of layered composite SMA plates obtained as staking sequence of thin layers, some of which contain SMA wires. A multiscale approach is proposed to model the mechanical behavior of SMA composites.

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