Georges Griso

Simplification of the elasticity equations in a structure made of rods

We present a new decomposition of rods structure displacements and then simplifications in the strain tensor. We apply these results to obtain error estimates in the three-dimensional linearized elasticity problem posed in a structure made of rods.

Any displacement u of a rods structure is the sum of an elementary displacement U_e (generalization of the Bernoulli-Navier displacements) and a residual displacement $\overline{u}$. Using U_e, we get informations about the displacements of the rods middle lines and about the cross-sections rotations. The displacement $\overline{u}$ expresses the warping of rods cross-sections.

We introduce an unfolding operator $\mathcal{T}_\delta$: with a dilatation we transform each rod in order to obtain fixed domains. In any reference rod, we suggest to eliminate the partial derivatives with respect to x_3 (the direction of the middle line) of the warping $\overline{u}$ to simplify the unfolded strain tensor. So we obtain a formal strain tensor of the unfolded displacement and a space of formal displacements $\mathcal{D}^f$ of the reference rods.

We consider the classical elasticity problem posed in a structure made of rods. Then we set the formal elasticity problem in $\mathcal{D}^f$. We prove that we must solve a problem posed in the set of all middle lines of the rods. This variational problem is verified by the middle lines displacements and the rotation angles of the cross-sections.

Eventually we give the distance between the unfolded displacement solution of the initial problem and the solution of the formal problem. The techniques leading to these estimates are the same as those presented in [3] and [4] for the unfolding method in periodic homogenization.

[1] G. Griso, Asymptotic behavior of curved rods by the unfolding method, *Math. Methods Appl. Sci.* **27** (2004), 17, 2081-2110.

[2] G. Griso, Décomposition des déplacements d'une poutre : simplification d'un problème d'élasticité, *Acad. Sci. Paris Mécanique* (2005), to appear.

[3] G. Griso, Error estimate and unfolding for periodic homogenization, *Asymptotic Anal.* **40** (2004), 3-4, 269-286.

[4] G. Griso, Interior error estimate for periodic homogenization, *C. R. Math. Acad. Sci. Paris* **340** (2005), 3, 251-254.